The Shallow Water Wave Ray Tracing Model SWRT

Prepared for: BMT ARGOSS
Our reference: BMTA-SWRT
Date of issue: 20 Apr 2016

Document Status Sheet

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Contents

Tables

Figures

1. Introduction

1.1. Model environments

The wave ray method is typically used for preliminary assessment of nearshore wave conditions: you can quickly obtain a reasonable estimate of the nearshore wave conditions for relatively low costs. At BMT ARGOSS, the wave ray model is used in both hindcast and forecast mode. In hindcast mode the model is used to assess the nearshore wave climate based on translation of 20+ years of offshore time series of wind and wave data. Offline, the model is run by consultants (referred to as consultancy mode). The wave ray model is also part of the online waveclimate.com service. In operational forecast mode, nearshore wave conditions are mostly predicted seven days ahead four times a day.

In operational forecast and under waveclimate.com the SWRT core is called and fed with data in an automated way. In addition to the nearshore location, the forecaster/user enters/clicks on an offshore location to define the extent of the offshore boundary (a rectangular frame) for SWRT. This offshore frame is found automatically by the application and the offshore data are passed to SWRT for translation to the nearshore site. The wave conditions are provided as wave spectra at all grid points on the offshore frame. The wind data, to be used by SWRT for local growth from land and boundary, are copied from the grid point (not necessarily on the frame) closest to the nearshore site.

In consultancy mode, the consultant explicitly sets the offshore frame and the set of offshore points on that frame. Wind data used for local nearshore growth will be copied from the nearest offshore point, possibly but not necessarily on the frame.

Boundary data are calibrated with satellites under waveclimate.com (auto-correction for the normal climate; this removes any systematic error) and for consultancy (full manual calibration with special attention for the more severe values). In forecast mode, boundary conditions are still un-calibrated. We intend to also add calibration in forecast mode.

1.2. The model in a nutshell

The ray tracing model translates offshore wave conditions to the nearshore location of interest. Offshore wave spectra are taken from (deep water) model grid points along a rectangular boundary. Nearshore wind data are copied from the nearest model grid point.

The near shore wave transformation takes the following processes into account:

- **Sheltering** of certain wave directions by island and capes
- **Refraction** of waves by varying bathymetry
- **Shoaling** of waves due to changing water depth
- **Local wave growth,** fetch-limited both from land and from the seaward boundary
- **Depth induced breaking** of waves due to the limited water depth
- **White-capping,** wave breaking due to steepness in deep water

The first step in the wave transformation is to calculate the wave trajectories which end at the near shore location of interest. A wave ray is the trajectory in space followed by a wave packet with a particular frequency and initial propagation direction. It is determined by the spatial variation of depth and current. The present model only accounts for the effect of depth variation; the effect of current variation is ignored. A wave ray bends (‘wave refraction’) where there is a gradient in depth perpendicular to the wave direction, that causes a gradient in propagation speed along a wave crest.

Starting from the near shore location, ray back-tracing is used to compute the wave rays for a discrete set of frequencies and nearshore propagation directions. The frequencies are the same as those of the wave model providing the offshore wave conditions and the wave directions are on a regular grid of 2.5 degree spacing. A standard differential equation solver is used to solve the coupled equations for position and propagation direction backwards while deriving the wave number magnitude from the local depth. The depth is determined from a realistic digital bathymetry of the region of interest of sufficient resolution that was derived from nautical charts. The ray curvature is limited by an uncertainty principle to prevent excessive directional variation. The solver adapts its step length to guarantee sufficient accuracy. Wave rays either end at land or at the rectangular offshore boundary (the depth chart extends beyond the offshore boundary).

Incoming components of the boundary wave spectra are propagated along the rays towards the location of interest. For paths traced back to the shore, fetch-limited growth is estimated from the local wind to account for waves from these directions. Where appropriate (this depends on the wind-sea component already present at a boundary point) fetch-limited growth is also added for paths traced back to the seaward boundary. Finally the propagated and fetch-limited wave spectra are combined at the nearshore location of interest and wave energy is reduced to satisfy the breaking criteria when necessary (based on the depth at the nearshore location).

Provided that the wave ray model has been configured correctly, the quality of the nearshore translation mainly depends on

- The accuracy of the bathymetry
- The choice of the offshore boundary
- The calibration of the offshore wave conditions (not in forecast mode yet)

1.3. Example model configuration off Panama

For illustration purposes and for reference in the remainder of this document, an example configuration of the wave ray model for a nearshore location off Panama is provided. The following plots summarize the model configuration:

- Figure 1- Nearshore location, rectangular offshore boundary active offshore grid points
- Figure 2- Wave rays used for propagation of waves from the offshore boundary
- Figure 3- Large scale bathymetry and offshore boundary
- Figure 4- Small scale bathymetry for the nearshore site

Remarks:
The wave rays shown in Figure 2 trace back to the offshore boundary; (only) these wave rays are used for propagation of waves from the boundary to the nearshore location. The red dots in Figure 2 mark points shallower than the nearshore depth. Propagating waves may break over these shallows before arriving nearshore. This is not accounted for by the model which only *applies wave breaking at the nearshore location after wave propagation* (see wave breaking section 2.6 for further detail).

Large scale bathymetry plots (Figure 3) show the offshore bathymetry chart; they are typically hundreds of kilometres wide with step size 1 km. Small scale plots (Figure 4) zoom in on the nearshore area around the location of interest; these plots normally span 10-20 km with resolution 100 m.

Note that not all points at the boundary frame shown in Figure 1 have been selected to provide offshore wave conditions: location 10N, -80.5E was apparently skipped by the consultant. In forecast mode and under waveclimate.com all model points at the frame will be used for provision of wave conditions.

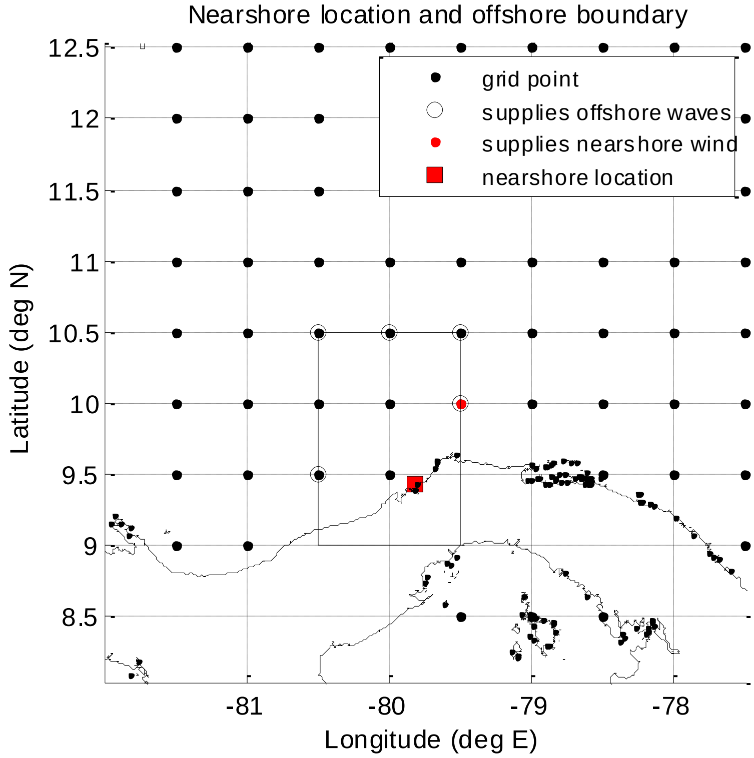

Figure 1 Model configuration off Panama with 5 offshore points (hindcast consultancy mode)

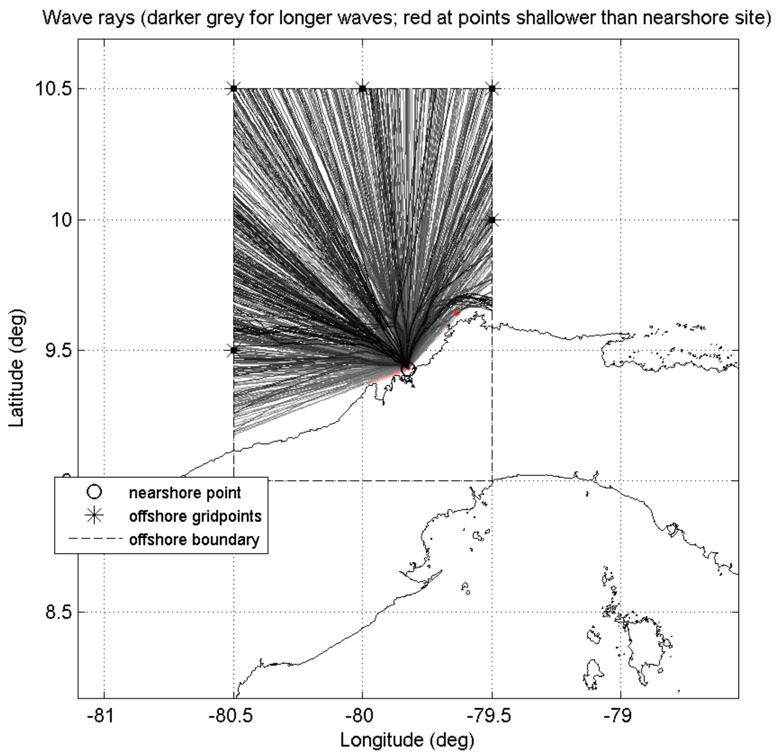

Figure 2 Wave rays for wave propagation from the boundary off Panama

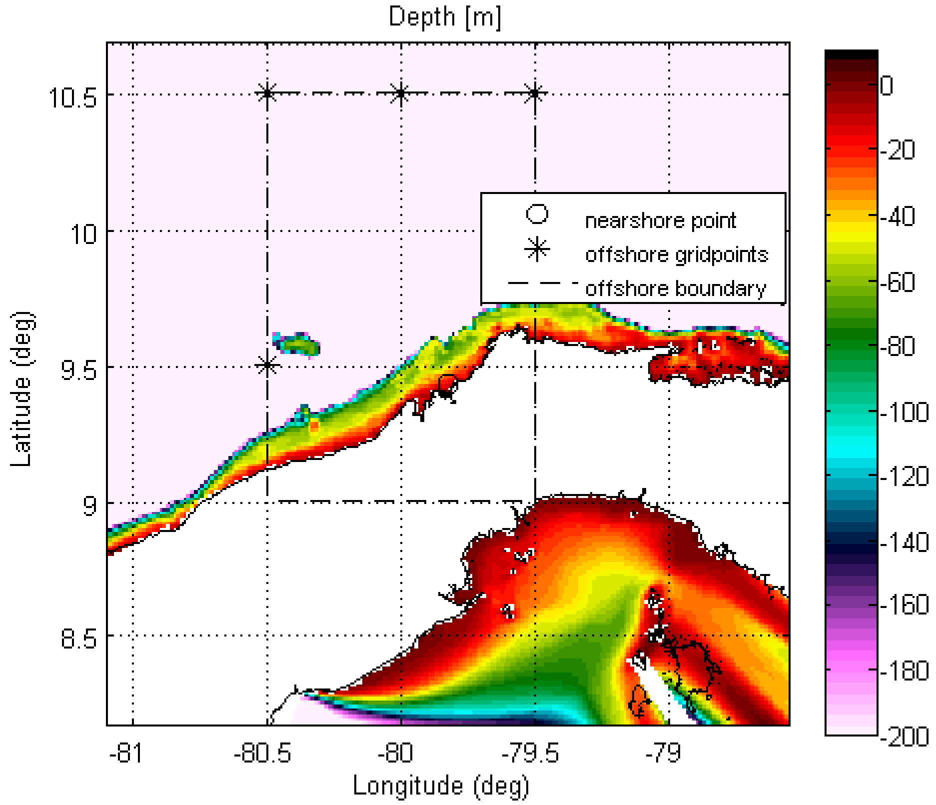

Figure 3 Large scale gridded bathymetry and offshore boundary off Panama

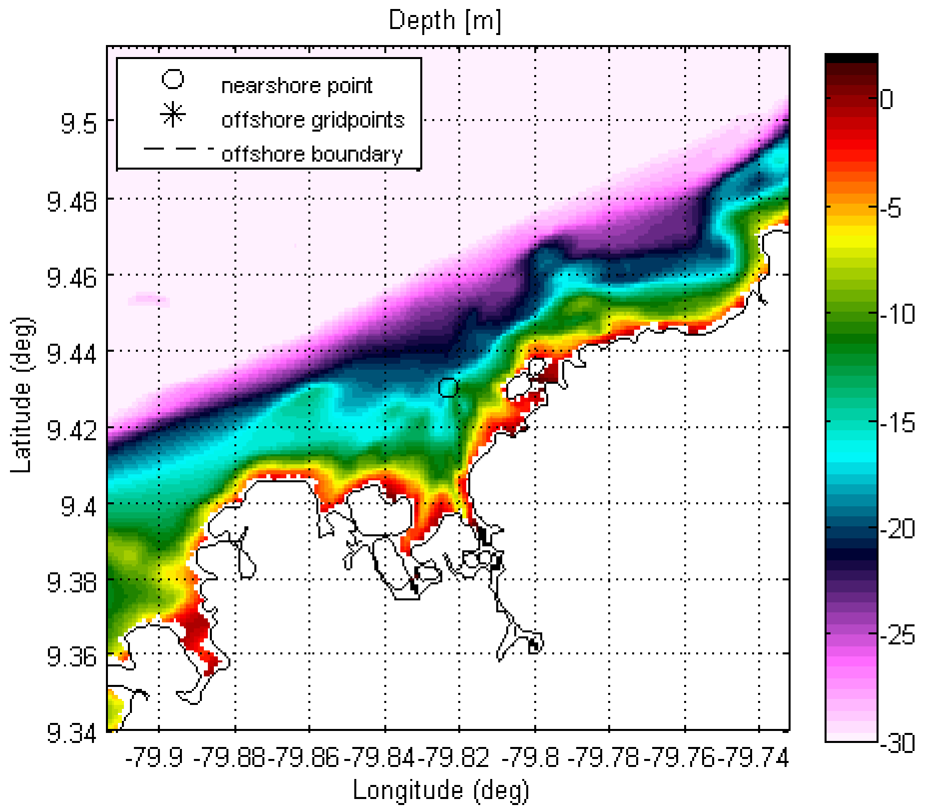

Figure 4 Nearshore gridded bathymetry off Panama (detail)

2. Methods and formulations

In the next sections, formulations are provided for the processes taken into account by the wave ray model (for the processes listed in section 1.2).

2.1. 1D and 2D wave spectra

The wave ray model can translate both 1D and 2D wave spectra. 1D or quasi 2D wave spectra comprise three 1D spectra, i.e. energy density (E), mean direction (Θ_m) and directional spread (S) as a function of spectral frequency. Real or full 2D spectra hold energy density as a function of spectral frequency and spectral direction. The resulting wave spectra at the nearshore location of interest can also be 1D or 2D.

Full 2D wave spectra provided at the offshore points vary with time (t), spectral frequency (f), spectral direction (θ) and offshore location (p):

$$E(t, f, \theta, p)$$

For 1D or quasi 2D spectra, the spectral direction disappears:

$$E(t, f, p) \quad \theta_m(t, f, p) \quad S(t, f, p)$$

Full 2D wave spectra can be reconstructed from the quasi 2D spectra by assuming a cosine-squared directional distribution energy density (E) based on the directional spread (S) around the mean wave direction (Θ_m) for a given frequency.

For brevity, the time (sea state) dimension is left out for the wave spectra in the following sections.

2.2. Bathymetry

Water depth, i.e. the height of the water column, used by the wave ray model is computed as water level minus seabed level. The seabed level in the area of interest is based on the underlying nautical chart (chart datum is LAT-Lowest Astronomical Tide). The water level is the deviation of the water surface relative to LAT. By default, the water level is set to zero. The water level is a constant applied to the whole offshore area. In areas with significant tidal effects, the water level can be used to compensate for the difference between MSL (Mean Sea Level) and LAT or even for the difference between HAT (Highest Astronomical Tide) and LAT.

Figure 3 and Figure 4 show the gridded seabed level (negative values) for the example area off Panama. With water level set to zero by default, water depth (positive values) is equal to minus the seabed. So, despite the title, the colour bar in these figures indicates seabed values rather than depth. The plotted depth charts extend beyond the offshore boundary but depth values outside the rectangular boundary do not affect the nearshore results as wave rays end at the boundary. So any peculiar depth issues outside the offshore boundary can safely be ignored.

The nautical charts, digitized into sets of depth values at isolated points, have varying spatial extent and resolution. The effective accuracy level available depends on the area of interest. Alternative/additional sets of point-oriented depth values can be used/added if available, for example depth measurements obtained from the client. The wave ray model merges the sets of isolated points from the various accuracy levels (low resolution samples are replaced by nearby high resolution samples) and combines the merged set into smoothed, gridded depth values.

In the plots below, the depth values at the original set of isolated points (colour-filled black circles) have been plotted on top of the resulting gridded bathymetry. Figure 5 shows the original data points and depths on top of the large scale depth chart (also seen in Figure 3). Figure 6 provides more detail for the nearshore area (also in Figure 4).

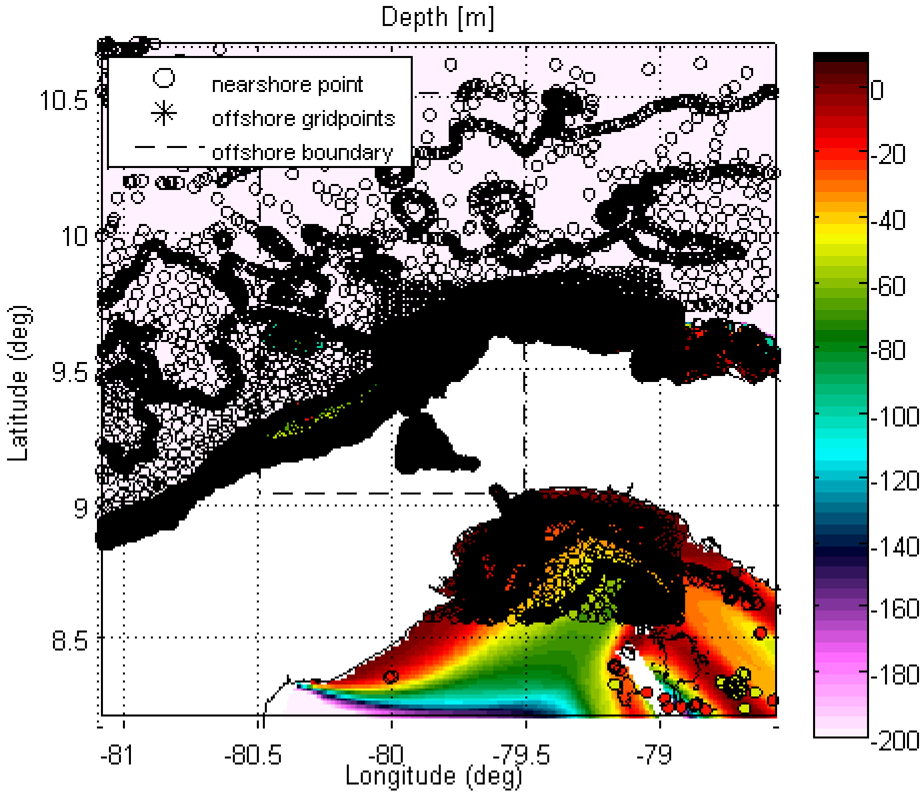

Figure 5 Large scale gridded bathymetry and original isolated points off Panama

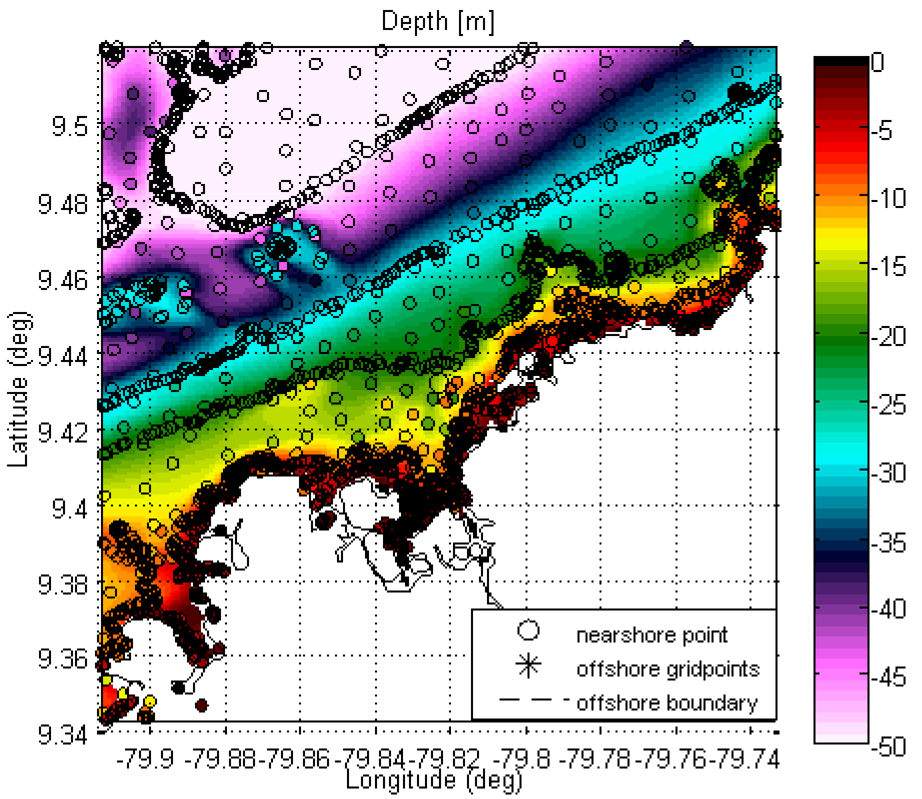

Figure 6 Nearshore gridded bathymetry and original isolated points off Panama

The effective resolution of the resulting depth chart depends on the density of the original soundings. In general, the resolution of the gridded depth in the nearshore area is about 100 m (step size used for Figure 6) whereas the resolution further offshore is roughly 1 km (step used in Figure 5).

The gridding of the merged soundings is done in two steps. For the large scale area (Figure 3, typically hundreds of kilometres wide), a polar grid is used with the pole close to the nearshore location. In this way resolution increases towards the nearshore location. For the small scale nearshore area (Figure 4; typically tenths of kilometres wide) gridding is refined in a local Cartesian grid and results are inserted back into the polar grid.

The gridding of the soundings, both for the large scale polar grid and for the small scale Cartesian grid, is a multiple-step process in itself. A multi-grid smooth algorithm is applied in order to minimize the residuals between the soundings and the estimated depth values on multiple grids with increasing resolution inside the given area. Typically the number of grid levels equals the number of accuracy levels of the original soundings. The grid resolution doubles between consecutive levels up to the desired resolution of the finest grid. The algorithm is similar to the four-colour Gauss-Seidel method with colours corresponding to grid levels. For each grid level, the minimization starts with the coarser solution found one level up (if any).

World-wide coastlines come from an independent data set, i.e. GSHHS (Global Self-consistent Hierarchical High-resolution Shorelines). In case of inconsistency, the coastlines may be overridden by the bathymetry, i.e. coastline nodes can be replaced by the nearest point with zero depth.

2.3. The dispersion relation and propagation velocities

The dispersive character of propagating waves (longer waves travel faster than shorter waves) is contained in the dispersion relation. This basic relation from linear wave theory provides a unique relation between (radian) wave frequency and wave number for given depth:

$$\omega^2 = gk\tanh(kd)$$

Eq. 2.3-1 Dispersion Relation

The phase speed of the waves (propagation speed of a wave crest) and the group velocity (propagation speed of the wave energy) are defined as

$$c = \omega / k \qquad c_g = \partial\omega / \partial k$$

Eq. 2.3-2 Phase speed and group velocity

On deep water, wave energy travels with half the speed of a wave crest. With decreasing depth, the phase speed also decreases whilst the group velocity increases: the difference between the two gradually disappears. This behaviour follows from the ratio of the two:

$$\frac{c_g}{c} = n = \frac{1}{2}\left(1 + \frac{kd(1-\tanh^2(kd))}{\tanh(kd)}\right) = \frac{1}{2}\left(1 + \frac{2kd}{\sinh(2kd)}\right)$$

Eq. 2.3-3 Ratio of group velocity and phase speed

In Eq. 2.3-1 to Eq. 2.3-3 :

ω is radian wave frequency ($\omega = 2\pi f$), k is wave number, d is depth

c_g is the group velocity of the waves, c is the phase speed and n is their ratio.

2.4. Wave rays

Due to depth variations along the crest, with a corresponding variation in phase speed, waves slowly change direction as they approach the coast: the waves turn towards the region with shallower water. Off Panama for example (see Figure 2), the wave ray pattern indicates that longer waves travelling parallel to the coast from east to west would gradually change direction to arrive at the nearshore location from the northeast.

By means of the reverse refraction technique, also referred to as back tracing, the trajectories (in terms of wave direction and easting and northing co-ordinates; easting taken towards the coast) of the wave rays fanning out from the nearshore location are found for all nearshore directions (step 2.5 degrees) and all spectral frequencies:

$$\frac{\partial\theta}{\partial n} = +\left(\frac{\partial d}{\partial x}\cos(\theta+\frac{\pi}{2}) + \frac{\partial d}{\partial y}\sin(\theta+\frac{\pi}{2})\right)\cdot\frac{\omega}{2c_g}\left(\frac{1}{\tanh(kd)} - \tanh(kd)\right)$$

$$\frac{\partial x}{\partial n} = -\cos\theta \quad \frac{\partial y}{\partial n} = -\sin\theta$$

Eq. 2.4-1 Wave ray curvature

In Eq. 2.4-1:

The spatial rate of change in wave direction $\partial\theta$ per unit forward distance ∂n is equal to the curvature of the wave ray; c_g is the group velocity of the waves, c is the phase speed; θ is local wave direction (equal to the direction of the wave ray; going to, relative to easting and positive anti-clockwise); d is depth; n , m are co-ordinates along the wave ray and the wave crest respectively and x , y are easting and northing co-ordinates ($x = n\cos\theta$; $y = n\sin\theta$).

To ensure smoothly curved wave rays, curvature is maximized as follows:

$$\frac{\partial\theta}{\partial n} \le \frac{k}{2\pi} = \frac{1}{L}$$

Eq. 2.4-2 Maximum wave ray curvature

It is important to realize that the wave ray trajectories are *potential* paths for propagation of offshore wave energy to the nearshore site: the wave ray pattern solely depends on the fixed bathymetry and the coastline. Although depth in the coastal area can be corrected with a constant (to simulate HAT or MSL instead of default LAT), the model does not account for water level changing over time (also see section 3.4 on limitations). Hence, the wave ray pattern is fixed prior to the computation of wave propagation per sea state (explained in section 2.5).

2.5. Wave propagation (sheltering, refraction and shoaling)

At the nearshore location, offshore wave energy can only arrive from nearshore directions for which at least one wave ray (there is one wave ray for each combination of nearshore direction and spectral frequency) traces back to the offshore boundary (and not to land). Nearshore directions (step 2.5 degrees) having at least one wave ray tracing back to the offshore boundary are referred to as unsheltered directions. For unsheltered directions, the nearshore site is exposed to waves from open sea. Nearshore directions for which all wave rays trace back to land are sheltered for waves from open sea. Sheltering occurs in the vicinity of islands and capes. At the nearshore site off Panama for example (see Figure 2), ocean waves cannot arrive from the east because of sheltering.

Each wave ray is a *potential* path for propagation of wave energy from the offshore boundary, i.e. from the start of the wave ray, being the point of intersection between the wave ray and the offshore boundary frame.

The amount of energy actually arriving nearshore from a particular direction along a particular wave ray depends on the amount of wave energy present at the starting point of the wave ray in the related spectral frequency bin and travelling towards the coast in the initial direction of that particular wave ray. Obviously, most wave ray starting points do not coincide with an offshore boundary grid point: the wave energy at the start of the wave ray is copied from the appropriate spectral bin of the wave spectrum at the nearest offshore boundary grid point.

A weight factor is applied to the offshore energy in each spectral bin of the offshore wave spectra to model the shoaling and refraction effects between the offshore location and the nearshore location. Shoaling effects are determined by the ratio of offshore and nearshore group velocity whilst refraction depends on the ratio of offshore and nearshore wave length (or wave number).

Full 2D spectra

If full 2D wave spectra are transferred, nearshore 2D spectra are found as follows:

$$E_{near}(f,\theta) = \int_{\theta_{near,\theta}} E_{off}(f,\theta_{near},p_{off}) \cdot W(f,\theta_{near}) \frac{d\theta_{near}}{d\theta}$$

Eq. 2.5-1 Refraction and shoaling (full 2D spectra)

In Eq. 2.5-1:

E_{near} is the 2D spectral density at the nearshore location.

f is spectral wave frequency and θ is spectral wave direction.

θ_{near} is nearshore wave direction (step 2.5 degrees).

$\theta_{near,\theta}$ is the subset of nearshore directions (step 2.5 degrees) falling inside the spectral directional bin centred around θ (typical bin size is 30, 15 or 10 degrees).

E_{off} is the 2D spectral energy density (energy density at nearshore direction θ_{near} is found by linear interpolation in spectral directions θ) at the nearest offshore boundary grid point.

p_{off} is the nearest offshore boundary grid point i.e. the grid point nearest to the end of the wave ray associated with spectral frequency f and nearshore wave direction θ_{near}:

$$p_{off} = p_{off}(f, \theta_{near})$$

W is a weight factor applied to offshore wave energy, incorporating shoaling (ratio of offshore and nearshore group velocity) and refraction (ratio of offshore and nearshore wave length) effects:

$$W(f, \theta_{near}) = \frac{c_{g,rayend}(f, \theta_{near})}{c_{g,near}(f, \theta_{near})} \frac{L_{rayend}(f, \theta_{near})}{L_{near}(f, \theta_{near})}$$

Eq. 2.5-2 Weight factors for refraction and shoaling

In Eq. 2.5-2:

$c_{g,rayend}$ is the offshore group velocity at the end of a wave ray

L_{rayend} is the offshore wave length at the end of a wave ray

$c_{g,near}$ is the group velocity at the nearshore location

L_{near} is the wave length at the nearshore location

Quasi 2D spectra

If quasi 2D spectra are to be translated, full 2D offshore spectra are simulated by assuming a cosine-squared directional distribution of offshore wave energy based on the directional spread around the mean wave direction (corrected for misalignment with the initial direction of the wave ray) for a given frequency. The nearshore quasi 2D spectra, i.e. nearshore 1D spectral wave energy density (Eq. 2.5-3), nearshore 1D spectral mean direction (Eq. 2.5-5) and nearshore 1D directional spread (Eq. 2.5-6) as a function of spectral frequency, are computed as explained below.

Nearshore 1D wave energy

Nearshore 1D wave energy density is found as:

$$E_{near}(f) = \int_{\theta_{near}} E_{off}(f, p_{off}) \cdot D_{off}(f, \theta_{near}, p_{off}) \cdot W(f, \theta_{near}) \cdot d\theta_{near}$$

Eq. 2.5-3 Refraction and shoaling (1D spectral wave energy)

In Eq. 2.5-3:

E_{near} is the 1D spectral energy density at the nearshore location.

E_{off} is the 1D spectral energy density at the nearest offshore boundary grid point.

p_{off} as in Eq. 2.5-1.

W as in Eq. 2.5-1 and Eq. 2.5-2.

D_{off} is the directional distribution of the wave energy at the nearest offshore boundary grid point corrected for misalignment with the direction of the wave ray:

$$D_{off} = \frac{\Gamma(s_{off}+1)}{\Gamma(s_{off}+0.5)\cdot 2\sqrt{\pi}} \left[\cos^2\left(\frac{\theta_{rayend}(f, \theta_{near}) - \theta_{m,off}(f, p_{off})}{2} \right) \right]^{s_{off}}$$

Eq. 2.5-4 Cosine-squared directional distribution of offshore wave energy

In Eq. 2.5-4:

θ_{rayend} is the direction of the wave ray at the offshore end of the wave ray, i.e. at the point of intersection of the wave ray and the offshore boundary.

$\theta_{m,off}$ is the 1D mean spectral direction at the nearest offshore boundary grid point.

s_{off} is a directional width parameter derived from S_{off} by means of a lookup table T:

$$s_{off} = s_{off}(f, p_{off}) = T(S_{off}(f, p_{off}))$$

S_{off} is the 1D spectral spreading at the nearest offshore boundary grid point.

Figure 7 shows the relation between directional width parameter s and spectral spread S based on lookup table T. Figure 8 shows the (normalized) directional distribution D for selected values of the directional width parameter s: the maximum occurs for the initial direction of the wave ray.

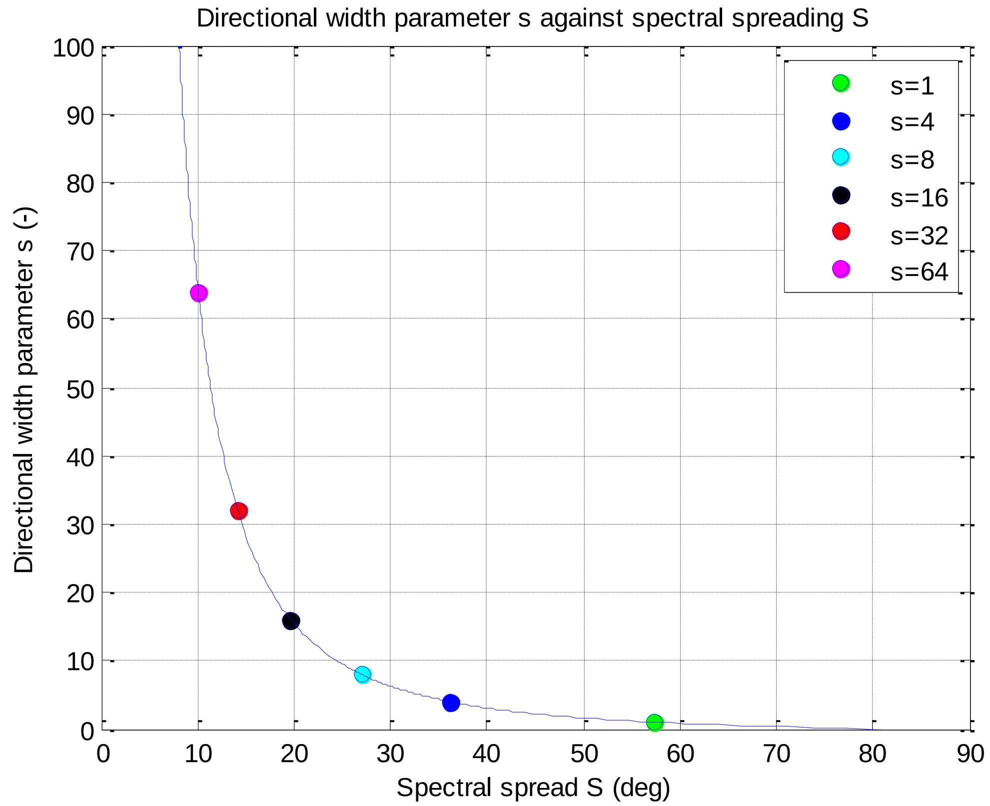

Figure 7 Directional width parameter s against spectral spread S

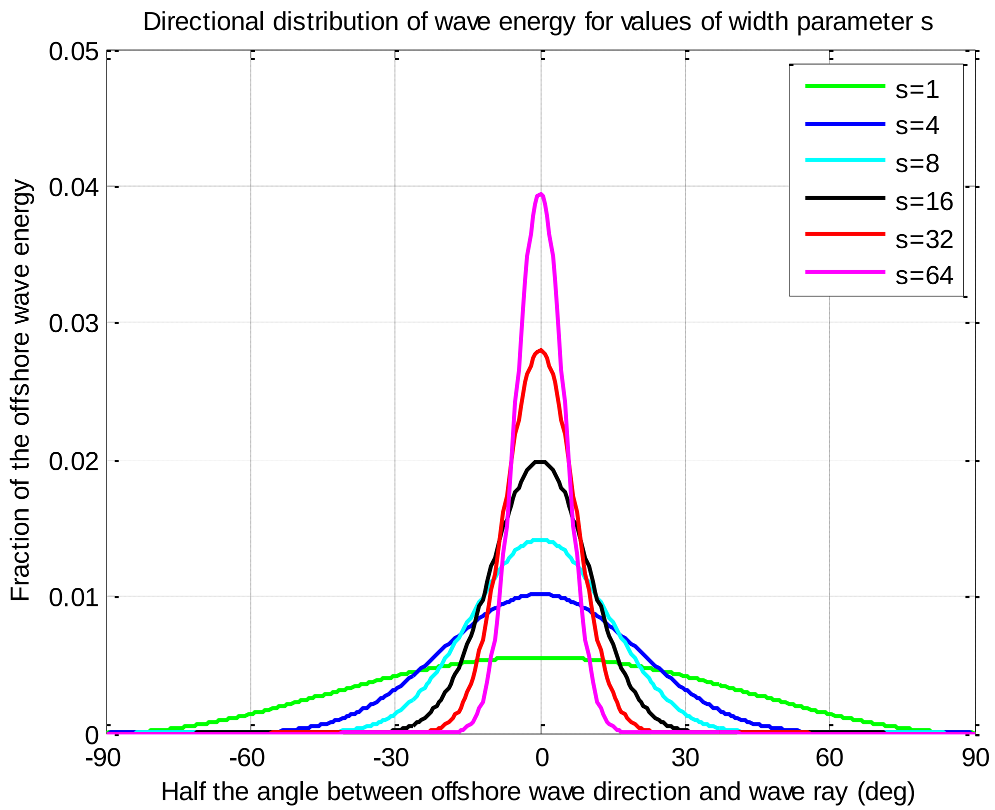

Figure 8 Directional distribution model of wave energy in a spectral frequency bin

Nearshore 1D spectral mean direction

Nearshore 1D spectral mean direction is found as:

$$\theta_{m,near}(f) = \arctan \frac{Y(f)}{X(f)}$$

Eq. 2.5-5 Refraction and shoaling (1D spectral mean direction)

In Eq. 2.5-5:

$\theta_{m,near}$ is the 1D spectral mean direction at the nearshore location.

$$Y(f) = \int_{\theta_{near}} E_{off}(f, p_{off}) \cdot D_{off}(f, \theta_{near}, p_{off}) \cdot W(f, \theta_{near}) \cdot \sin(\theta_{near}) \cdot d\theta_{near}$$

$$X(f) = \int_{\theta_{near}} E_{off}(f, p_{off}) \cdot D_{off}(f, \theta_{near}, p_{off}) \cdot W(f, \theta_{near}) \cdot \cos(\theta_{near}) \cdot d\theta_{near}$$

Nearshore 1D spectral spread

Finally, nearshore 1D spectral spread is found as:

$$S_{near}(f) = \sqrt{2 \cdot (1 - r(f))} \qquad r(f) = \sqrt{\left(\frac{Y(f)}{E_{near}(f)}\right)^2 + \left(\frac{X(f)}{E_{near}(f)}\right)^2}$$

Eq. 2.5-6 Refraction and shoaling (1D spectral spread)

In Eq. 2.5-6:

S_{near} is the 1D spectral spreading at the nearshore location.

E_{near} is the 1D spectral density at the nearshore location (see Eq. 2.5-3).

2.6. Wave breaking

The modelling of wave breaking, due to steepness or limited depth, depends on total significant wave height which is unknown along the wave rays. Therefore wave breaking is only computed at the nearshore site based on local depth (also see model limitations in section 3.4).

It is assumed that the wave breaking process conserves the spectral shape. Hence, the energy in the total nearshore wave spectrum, i.e. the wave spectrum resulting from wave propagation, can be reduced (equally for all spectral bins) as follows:

$$E_{near,breaking} = E_{near,prop} \cdot [\max(\left(\frac{\hat{H}_{m0}}{H_{m0}}\right)^2, 1)]$$

Eq. 2.6-1 Reduction of spectral energy due to wave breaking at the nearshore site

In Eq. 2.6-1:

$E_{near,pro}$ is the propagated energy density spectrum, either 1D or 2D.

$E_{near,breaking}$ is $E_{near,pro}$ reduced for wave breaking applied at the nearshore location.

$\hat{H}_{m0}$ denotes the maximum wave height computed as

$$\hat{H}_{m0} = \varepsilon \frac{2\pi}{k} \tanh\left(\frac{\gamma}{\varepsilon}\frac{k}{2\pi} d\right) = \varepsilon L \tanh\left(\frac{\gamma}{\varepsilon}\frac{d}{L}\right)$$

Eq. 2.6-2 Maximum wave height for wave breaking

In Eq. 2.6-2 wave breaking due to both white capping and limited depth are accounted for: ε is the maximum steepness in deep water (set to 0.055) and γ is the breaker index in shallow water (set to 0.55). The nearshore wave number is found by substitution of nearshore *mean* wave period and nearshore depth in the dispersion relation (Eq. 2.3-1).

2.7. Local wave growth

Local wave growth involves growth of waves due to wind blowing from land as well as from the open sea boundary. For growth with wind blowing from the open sea boundary, an initial condition for the wind-sea is determined from the boundary spectra. The computed wind-sea spectrum is not simply added to the propagated spectrum, but only a fraction which, when added to propagated energy in the same frequency range, would produce the amount of wind-sea energy computed by the growth curve. If the propagated boundary spectrum already contains a large enough wind-sea component, then local wave growth from open sea will not add/change much. For local growth with wind blowing from land, the initial wave energy is (of course) set to zero. In general, locally generated waves have a high frequency and are hence less affected by bathymetric effects than the longer waves from open sea.

For local growth, fetch lengths to the boundary or to land are simply measured along straight lines (not along the curved wave rays).

In Figure 9 and Figure 10 all wave rays are plotted, including the wave rays tracing back to land. In addition, the wave rays associated with the highest spectral frequency, also referred to as HF-rays, are highlighted (cyan). These HF rays are practically straight lines as shortest waves hardly refract. Therefore fetch length for local wave growth from the boundary or from land can be readily approximated by the distance between the nearshore location and the end of the HF wave rays at the boundary (blue dots) or near land (green dots).

Please note that Figure 9 and Figure 10 also show non-HF wave rays tracing back to land (black lines). These wave rays remain unused: only begin and end of the HF wave rays (cyan lines) are used to determine the fetch length for local growth from land.

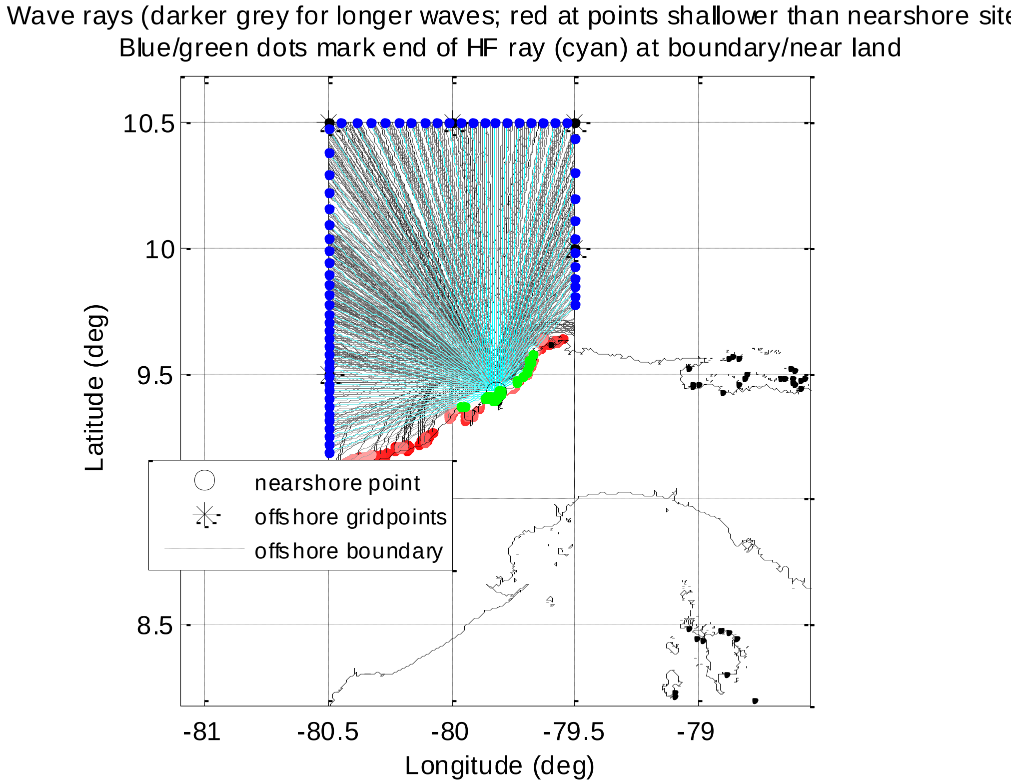

Figure 9 Local growth fetch (cyan) to boundary (blue dots) and land (green dots) off Panama

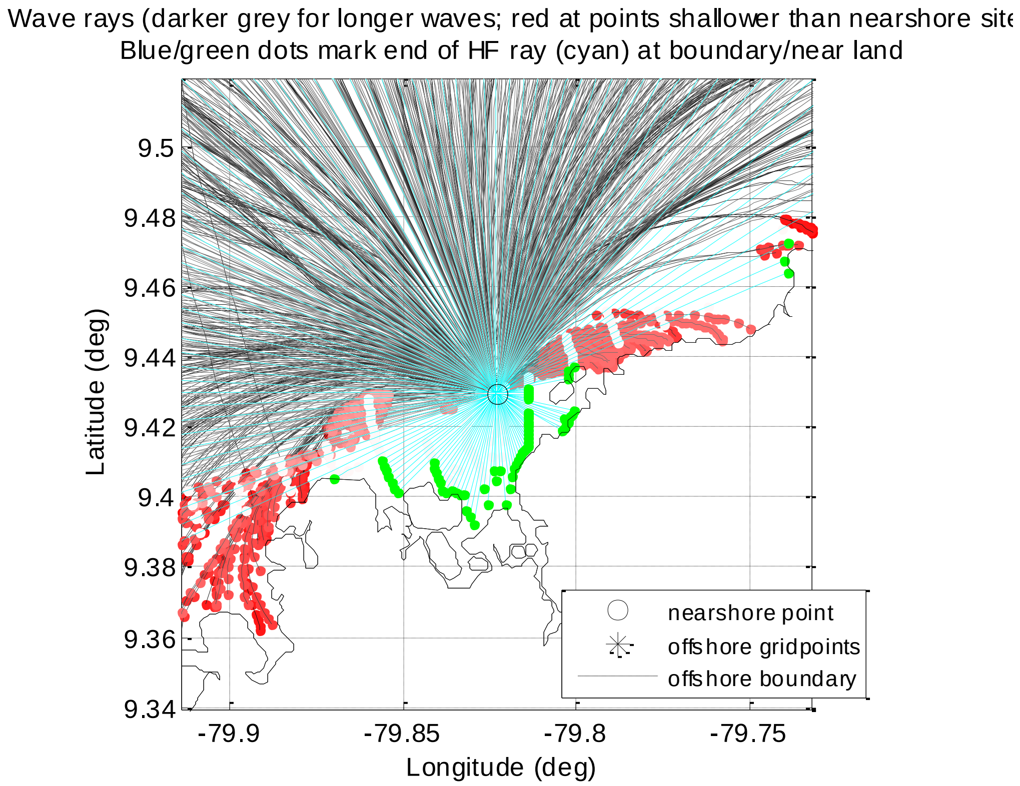

Figure 10 Local growth fetch (cyan) to land (green dots) off Panama (detail)

In the limited-depth waters relevant for the wave ray model, local wave growth depends on the nearshore wind speed, the fetch, the duration (the time elapsed since the wind started to blow) and the water depth. It is assumed that fetch and water depth are the limiting factors (not duration). Following Young and Verhagen (modified by Breugem and Holthuijssen) *dimensionless* significant wave height and *dimensionless* peak wave period evolve according to

$$\tilde{H}_{m0} = \tilde{H}_{\infty}\left[\tanh(k_3\tilde{d}^{m_3})\tanh\left(\frac{k_1\tilde{F}^{m_1}}{\tanh(k_3\tilde{d}^{m_3})}\right)\right]^p$$

$$\tilde{T}_p = \tilde{T}_{\infty}\left[\tanh(k_4\tilde{d}^{m_4})\tanh\left(\frac{k_2\tilde{F}^{m_2}}{\tanh(k_2\tilde{d}^{m_4})}\right)\right]^p$$

Eq. 2.7-1 Local wave growth

In Eq. 2.7-1:

$\tilde{H}_{m0}$ denotes dimensionless significant wave height found as $\tilde{H}_{m0} = gH_{m0}/U_{10}^2$ Dimensionless peak wave period $\tilde{T}_p$, dimensionless fetch $\tilde{F}$ and dimensionless water depth $\tilde{d}$ are found in a similar manner.

The next table summarizes the coefficients seen in **Eq. 2.7-1**:

Coefficient	Description	Value
$\tilde{H}_{\infty}$	PM deep-water fully developed dimensionless H_{m0}	0.24
k_3	Limited-depth correction H_{m0}	0.343
m_3	Limited-depth correction H_{m0}	1.14
k_1	Parameter for transition of young to fully developed sea state H_{m0}	$4.41\text{x}10^{-4}$
m_1	Parameter for transition of young to fully developed sea state H_{m0}	0.79
p	Extra parameter for transition of young to fully developed sea state H_{m0}	0.572
$\tilde{T}_{\infty}$	PM deep-water fully developed dimensionless T_p	7.69
k_4	Limited-depth correction T_p	0.10
m_4	Limited-depth correction T_p	2.01
k_2	Parameter for transition of young to fully developed sea state T_p	$2.77\text{x}10^{-7}$
m_2	Parameter for transition of young to fully developed sea state T_p	1.45
q	Extra parameter for transition of young to fully developed sea state T_p	0.187

Table 1: Coefficients used in wave growth Eq. 2.7-1

For local wave growth, the wave ray model computes a time-independent table of fetches as a function of nearshore directions (step 2.5 degrees). Fetch length is computed as the distance between the nearshore location and the end of the wave ray for the highest frequency (HF) for a particular nearshore direction (HF wave rays are the cyan lines in Figure 9 and Figure 10). Figure 11 shows the resulting fetch length (red line) against nearshore direction for the example off Panama.

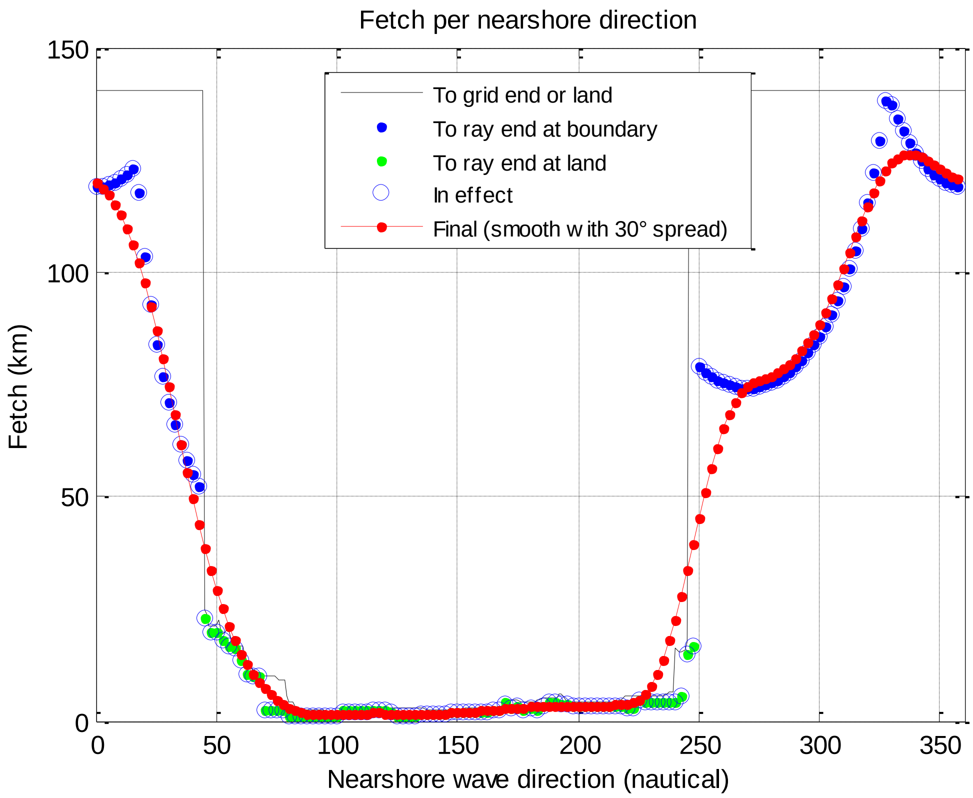

Figure 11 Fetch to boundary (blue dots) or to land (green) against nearshore direction off Panama

For each sea state, nearshore significant wave height and peak wave period are found from nearshore wind speed, fetch length (either to land or to the boundary depending on the direction of the nearshore wind in that sea state) and local depth according to Eq. 2.7-1. Per sea state, the actual fetch is interpolated from the time-independent fetch table by means of the nearshore wind direction. For growth from the offshore boundary, fetch is extended in order to include the wind-sea component already at the boundary, i.e. the wind-sea component (reduced for misalignment with the nearshore wind direction) at the boundary point nearest to the end point of the relevant HF wave ray. Using the inverse of Eq. 2.7-1, the additional fetch is found as the fetch that would have produced the reduced wind-sea for given nearshore wind speed and nearshore water depth.

From the resulting wave height and peak period, a nearshore wind-sea spectrum is created (1D energy density as a function of spectral frequency). The corresponding 1D spectral mean wave direction is set to the direction of the nearshore wind and the 1D spectral spreading is set to a typical constant value for wind-sea (corresponding to a spectral width parameter value 15- see Figure 8). Again, full 2D wind-sea spectra can be derived from the quasi-2D wind-sea spectra by assuming a cosine-squared directional distribution of wave energy based on the directional spread around the mean wave direction per frequency bin.

Per sea state, the total nearshore energy density spectrum, either 1D or 2D, is found by adding (part of) the energy from this wind-sea spectrum to the propagated wave spectrum. The amount of wind-sea added to a particular spectral bin depends on the wind-sea already present in the propagated wave spectrum in that bin:

$$E_{tot} = E_{pro} + E_{add} = E_{pro} + [1 - \left(\frac{H_{m0pro}}{H_{m0sea}} \right)^2] \cdot E_{sea}$$

Eq. 2.7-2 Combining propagated and local growth energy density spectra

In Eq. 2.7-2:

E_{tot} is the total 1D or 2D energy density spectrum at the nearshore location.

E_{pro} is the propagated 1D or 2D energy density spectrum at the nearshore location.

E_{add} is the 1D or 2D energy density spectrum added to E_{pro} (to account for local growth).

E_{sea} is the 1D or 2D energy density spectrum based on local growth at the nearshore location

H_{m0pro} is the wind-sea already present in E_{pro}.

H_{m0sea} is the wind-sea in E_{sea}.

For quasi-2D spectra, the 1D spectral mean direction and the 1D spectral spreading are computed as:

$$\theta_{m,near}(f) = \arctan \frac{E_{pro}(f).\sin(\theta_{m,pro}(f)) + E_{add}(f).\sin(\theta_{m,add}(f))}{E_{pro}(f).\cos(\theta_{m,pro}(f)) + E_{add}(f).\cos(\theta_{m,add}(f))}$$

Eq. 2.7-3 Combining propagated and local growth spectral mean direction

$$S_{near}(f) = \sqrt{E_{pro}(f) S_{pro}^2(f) + E_{add}(f) S_{add}^2(f)}$$

Eq. 2.7-4 Combining propagated and local growth spectral spreading

In Eq. 2.7-3 and Eq. 2.7-4:

E_{pro} is the propagated 1D energy density spectrum at the nearshore location.

E_{add} is the 1D energy density spectrum added to E_{pro} (to account for local growth).

$\theta_{m,near}$ is the resulting 1D spectral mean direction at the nearshore location.

$\theta_{m,pro}$ is the 1D spectral mean direction of the propagated waves.

$\theta_{m,add}$ is the 1D spectral mean direction of the wind-sea added for local growth.

S_{near} is the resulting 1D spectral spreading at the nearshore location.

S_{pro} is the 1D spectral mean direction of the propagated waves.

S_{add} is the 1D spectral mean direction of the wind-sea added for local growth.

3. Model configuration

3.1. Choice of the offshore boundary

- The boundary should be representative for the offshore wave conditions, so preferably in relatively deep water (depth in comparison to the wave length; see rules of thumb in section 3.3) and not too close to the shore. On the other hand, the boundary should not be too far away to avoid propagation of waves along wave rays over very long distances instead of wave propagation on a grid (as done in the offshore wave model). It is advised to keep the distance between the nearshore location and the offshore boundary between 25 and 200 km.
- Let the boundary rectangle ‘fence off’ the nearshore location from open sea and let the rectangle run across land ‘behind’ the nearshore location where possible (to ensure local growth from land where appropriate)
- Try to cover the whole rectangle with offshore model grid points, at least the part of the boundary where most of the waves arrive from open sea. If the offshore climate is homogenous, you might decide to leave out a few grid points in order the limit the amount of work to retrieve and calibrate the wind and wave data at the offshore boundary. Off Panama for example, one grid point was left out at the western boundary (see Figure 1 or Figure 2).
- Fore offshore conditions you can use but not solely rely on grid points at the grid boundary along the coast. Waves at model grid points are computed with a depth averaged over the grid cell. For the points at the grid boundary the resolution is probably too coarse to capture all wave dynamics. Therefore always try to include internal (with 4 neighbours) offshore model grid points so that the boundary conditions for the wave ray model are taken from locations with a smoother bathymetry.
- At the starting point of the wave rays at the offshore boundary, wave spectra are copied from the nearest offshore boundary grid point.

3.2. Other considerations

- The quality of the nearshore results most strongly depends on the quality of the offshore wave data and the accuracy of the bathymetry. Decent calibration of the offshore wave conditions with satellites is therefore essential.
- Under waveclimate.com, offshore hindcast wave spectra are automatically calibrated with satellite measurements with focus on the normal conditions, i.e. removal of the systematic error. Although auto-calibration also improves the above-average wave heights in general, we normally pay extra attention to calibration of the tail of the distribution for consultancy projects. Satellite-based hindcast calibration is also more complicated in the vicinity of land. This is another reason not to only rely on coastal wave model grid points at the boundary of the wave model grid.
- Despite the disadvantages of offshore points at the grid boundary mentioned above (1st and 2nd point in section 3.2; 4th bullet in section 3.1) it seems unwise to remove them altogether because they may well add valuable information on the wave conditions closer to the coast. For example on the direction of the waves. This seems a good reason to keep the southernmost boundary point in the Panama example (see Figure 1 or Figure 2).
- The water depth (height of the water column) used by the wave ray model is computed as water level minus seabed level. The seabed level is determined for the area inside the offshore boundary from the underlying nautical chart (chart datum is LAT). Please set water-level to

compensate for the difference between MSL and LAT (assumed datum). If the tidal range is substantial in comparison to the depth in the area, adapt the water-level to account for the tidal effect. The water-level is a constant elevation of the surface for the area inside the offshore boundary.

- The default seabed-level at the nearshore location is taken from the underlying nautical chart (with datum LAT). The user does have the option to override the default seabed-level at the nearshore location: the wave ray model will smooth the surrounding chart values to match the seabed-level provided by the user. Please be aware of the fact that a large difference between the default bed-level and the user-defined value will lead to an unrealistic seabed profile with an isolated high or low value.
- The plotted depth charts extend beyond the offshore boundary but depth values outside the rectangular boundary do not affect the nearshore results as wave rays end at the boundary. So any peculiar depth issues outside the offshore boundary can safely be ignored.
- Please note that wave rays are **potential** paths for propagation of wave energy; the wave rays are computed without any knowledge of the offshore climate. Hence, part of the wave ray pattern may turn out to be irrelevant in view of the governing offshore wave direction.

3.3. Rules of thumb to estimate shallow water effects

Here are some rules of thumb that often proved to be helpful to estimate the effect of the various shallow water effects taken into account by the wave ray model:

- Waves start 'to feel' the seabed if depth D is less than half the offshore wave length L0: D<0.5*L0. So incoming waves with 10 second wave period and wave length of about 156 m (L=1.56*T^2) start to refract and shoal for depths below 78 m. See Figure 12.
- Due to shoaling, offshore wave height first decreases to 90% of the deep water value H0 (reached at D=0.15*L0), then increases again to H0 (reached at D=0.05*L0) to reach values up to 2*H0 in shallower water. So for waves with offshore wave period of 10 s, the wave height minimum is reached at about 23 m water depth and waves start to exceed the deep water value H0 as of about 8 m. See red curve Figure 12 and Figure 13.
- Waves break due to steepness if H/L>0.14. Waves break due to limited depth if H/D>0.7 (breaker index). The breaker index depends on bottom slope and wave steepness and varies between 0.5 and 1.5. So with depth of 10m at the nearshore site waves higher than 5m start to break. See Figure 13.
- For a typical configuration, the spatial resolution of the depth map is about 100m.

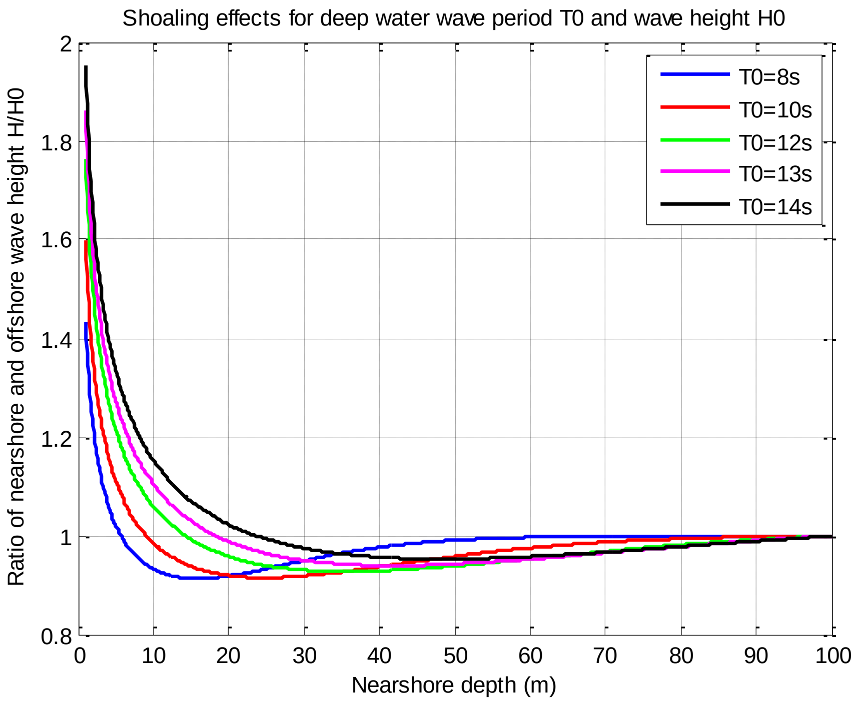

Figure 12 Theoretical shoaling effect against nearshore depth

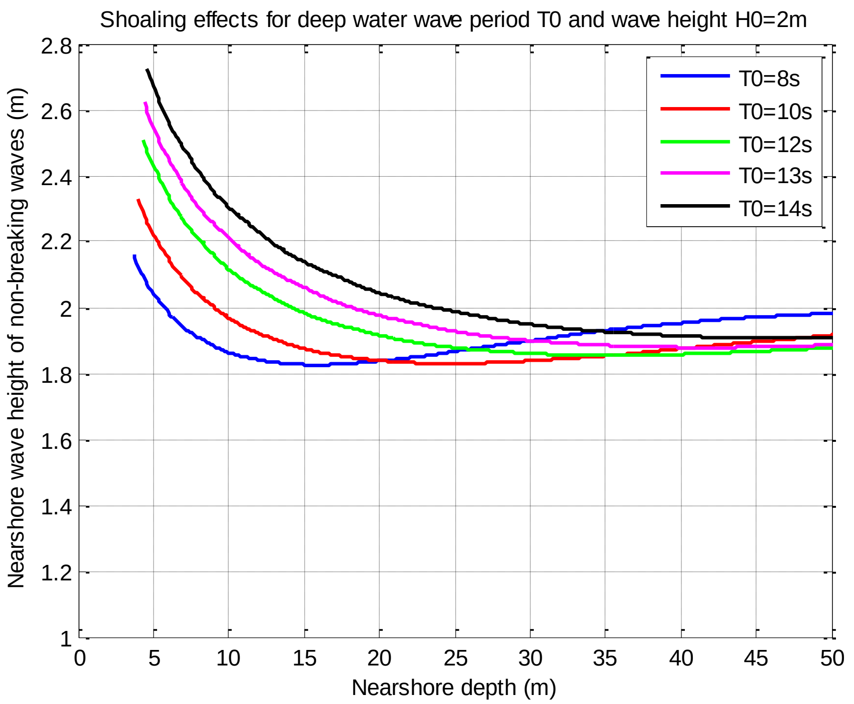

Figure 13 Shoaling effect against nearshore depth for non-breaking waves (breaker index 0.6)

3.4. Limitations and approximations

The wave ray method is typically used for preliminary assessment of nearshore wave conditions: with the wave ray model, you can quickly obtain a reasonable estimate of the nearshore wave conditions against relatively low costs. However, please be aware of the following limitations of the wave ray method (and consider using a more sophisticated model like SWAN instead if needed):

- The wave ray model **does not account for**: diffraction (obstacles/channels), reflection, currents, water level variation over time, bottom friction, wave breaking in the area (only at the nearshore location), non-linear wave-wave interaction (re-distribution of wave energy), 'smooth' wave propagation (the wave ray pattern is sensitive to steep/complex bathymetry). Diffraction is attended in more detail below.
- Ray tracing considers refraction but **not diffraction** of waves. Diffraction is interference of waves emanating from different points (e.g. points aligned parallel to an existing crest). When there is an obstruction (breakwater, steep seabed slope, such as dredged channel edge, et cetera). points on its boundary cannot act as sources in certain directions so the original symmetry of the propagating wave crests becomes perturbed. Effects are for example the spreading of waves behind a breakwater but also the trapping of waves in a dredged channel (forcing them to propagate along the channel axis), and in fact, more generally wave reflection on steep objects (including submerged ones). In case of 'objects' (whether submerged or not) which have scales in some direction which are smaller than a few wave lengths, diffraction can be important.
- **Wave breaking is only computed at the nearshore site using the local depth**, local significant wave height and wave period. It cannot be computed along the ray as we only propagate spectral components. Along a ray, significant wave height is unknown. To indicate that waves may have been broken before reaching the nearshore location, all points along rays at depths shallower than the nearshore depth are indicated with red dots in the wave ray plots (red dots along wave rays used for wave propagation shown in Figure 2). In principle, the ray tracing model is not applicable when waves are already breaking during propagation to the nearshore location, for example over sandbanks or reefs much shallower than the nearshore location. To check this, depth at the positions of red dots needs to be checked to see if substantial breaking may have occurred there.
- **Local wave growth** is computed from land boundaries and from the open sea boundary by default. Wave growth is **not computed along the wave rays** but by a simple growth curve using the nearshore depth and the fetch length to land or to the open sea boundary along a straight line in the wind direction. Wind data from the grid point nearest to the nearshore location are used to compute growth.
- The wave transformation depends heavily on the quality of the bathymetry used. In some places, the **resolution of the bathymetric data** is insufficient. It is important to check the charts computed in the model as well as available nautical charts and judge whether the computed charts are sufficiently accurate for the job. If not, then sometimes an output location further offshore could produce more robust results.
- In regions where the **bathymetry is complex and depth is only a few meters or less** at (some) points, the wave dynamics are difficult to model and the wave rays patterns may become unrealistic (rays become strongly curved). This includes regions where there are numerous islands, atolls or reefs between the offshore hindcast grid points and the nearshore location, for example west off Bermuda. The nearshore transformations in these regions perform poorly and should be treated with caution.

Appendix A – Frames of reference

Units and conventions

- Wind and wave directions are defined as “coming from” relative to true north positive clockwise.
- Unless explicitly stated otherwise, co-ordinates are expressed in degrees latitude and longitude, assuming a WGS84 co-ordinate system.
- Units are expressed using the SI convention if not stated otherwise:
  - Length or distance (wave height, surface elevation, water depth) in metres,
  - Time or duration (wave periods) in seconds,
  - Date and time are UTC/GMT,
  - Speed in metres per second,
  - Two-dimensional spectral energy density in m^2s/radian,
  - One-dimensional spectral energy density per frequency bin in m^2s,
  - Spectral variance in m^2,
  - Spectral wave frequency in Hz,
  - Spectral wave direction in radians (or degrees) positive clockwise from North,
  - Direction in degrees positive clockwise from North.

Metocean parameters

Waves

- **2D Spectral density of sea surface waves *E***
  The 2D spectral density describes how the variance of the sea surface elevation is distributed over spectral frequency f and spectral direction ɵ. It is referred to as the full 2D or the real 2D wave spectrum.

- **1D Spectral density of sea surface waves *E***
  The 1D spectral density describes how the variance of the sea surface elevation is distributed over spectral frequency f. It is often referred to as wave spectrum.

- **1D Spectral mean direction Θ_m**
  The spectral mean direction is the mean wave direction per spectral frequency bin.

- **1D Spectral directional spreading *S***
  The spectral directional spreading is the directional spreading of the waves per spectral frequency bin.

- **Quasi 2D wave spectrum**
  The set of the three 1D spectra above, i.e. 1D spectral density, 1D spectral direction and 1D spectral directional spreading are also referred to as quasi 2D spectra. The rationale behind this naming is the fact that a 2D spectral density can be reconstructed by means of these three 1D spectra by assuming a cosine-squared directional distribution (dependent on the spectral directional spreading) of the spectral density around the mean wave direction per frequency bin. **Buoys** that measure wave spectra provide quasi 2D spectra.

- **Spectral moment m_p**

For any integer p, m_p is the integral over frequency f of f^{p} multiplied by the wave spectrum, with f frequency in cycles per unit time. Remark: m_0 is the total variance of sea surface elevation.

- **Spectral frequency *f***
  Wave spectra are provided as a function of spectral frequency and spectral direction. Spectral frequency is equidistantly distributed on a logarithmic scale. Normally it ranges from 0.03 to 0.55Hz typically with 25 or 30 frequency bins.

- **Spectral direction θ**
  Wave spectra are provided as a function of spectral frequency and spectral direction. Spectral direction covers the full circle in radians, typically distinguishing 12, 24 or 36 directional bins.

- **Nearshore directions θ_{near}**
  The set of nearshore wave directions used to compute propagation of wave spectra to the nearshore site. The directional bin size is set to 2.5 degrees.

- **Wave height *H***
  This is crest-to-trough wave height of an individual wave (between two consecutive up-crossings of the still water level).

- **Mean directional spreading**
  This is the energy weighted mean directional spread of the total spectrum (***spreadd)*** or the directional spreading at the peak frequency (***spreadp***).

- **Significant wave height *Hs***
  Averaged wave height H of the 1/3 highest waves. Except on shallow water, *Hs* is accurately approximated by *Hm0*, defined as 4 times the standard deviation of the vertical surface displacement (4 times the square root of spectral moment m_0). In this report and in the accompanying time series file, we approximated significant wave height *Hs* by means of *Hm0*.

- **Principal wave direction *Hsd***
  The direction derived from the first-order directional Fourier moments (sine and cosine-weighted moments) of the directional wave spectrum. Wave direction is defined as “coming from”. It can also be defined for (a) limited range(s) of frequencies and represented as a function of frequency.

- **Wave periods based on spectral moments $Tm_{p,q}$**
  $Tm_{p,q} = (m_p / m_q)^{1/(q-p)}$ with m_p and m_q spectral moments, and p and q two distinct integers. Here, $Tm_{-1,0}$ and $Tm_{0,2}$, are referred to as mean wave period (***Tm***) and spectral mean zero-crossing wave period (***Tz***) respectively.

- **Zero-crossing wave period *T***
  Time elapsed between two consecutive up-crossings of the still water level.

- **Mean zero-crossing wave period *Tz***
  The average of the **zero-upcrossing period** *T* for a particular sea state. *Tz* is approximated by $Tz \approx Tm_{0,2}$ (see moment-based wave periods).

- **Peak wave frequency *Fp***
  This is the frequency where the wave spectrum reaches its maximum.

- **Peak wave period *Tp***
  The period corresponding to the frequency where the spectral density reaches its maximum.

- **Peak wave direction *Pd***
  This is the wave direction corresponding to the wave peak frequency.

- **Wave length *L***
  Wave length is the horizontal distance between two consecutive up-crossings of the still water level in the direction of wave propagation.

- **Wave steepness parameter s**
  A dimensionless parameter, defined as the ratio of significant wave height *Hs* to the deep-water wave length corresponding to the wave period $Tm_{-1,0}$, i.e., $s = (2\pi/g)\ Hs/(Tm_{-1,0})^2$. Gravitational acceleration g is taken equal to 9.81 m/s^2.

Wind

- **Wind speed *u10* and wind direction *u10d***
  Sustained wind speed at 10m above the (sea) surface and associated direction. Wind direction is defined as “coming from”. “Sustained” means averaged over 1 hour.

Tide

- **Tidal elevation and tidal datum LAT/HAT**
  Tidal elevation is the sea surface elevation due to astronomical tide relative to a fixed vertical datum, commonly mean sea level (MSL). Another commonly used vertical chart datum is Lowest Astronomical Tide (LAT), the height of the water at the lowest possible theoretical tide. Similarly, the height of the water at the highest possible theoretical tide is referred to as Highest Astronomical Tide (HAT).

Water depth and water level

- **Water depth**
  Water depth is the height of the water column (a positive number) relative to a fixed datum, commonly the mean sea level (MSL). However, unless stated otherwise, the wave ray model interprets water depth as taken relative to LAT.

- **Bathymetry**
  Bathymetry refers to the elevation of the surface of the Earth under water, i.e. elevation of the seabed.

- **Water level**
  Water level or sea surface level is found by adding the tidal elevation to the local water depth.